

OUNDLE SCHOOL

## MAIN ACADEMIC SCHOLARSHIP 2008

## MATHEMATICS

## PAPER 2

## 2 hours.

## CALCULATORS WILL BE NEEDED FOR THIS PAPER.

## INSTRUCTIONS TO CANDIDATES. <br> You are not expected to have time to do all the questions. <br> You may answer the questions in any order. <br> Choose those questions which you think you can answer best. <br> Remember to show your working and clearly show the method you are using. <br> Take $\pi$ as either 3.14 or the value on your calculator. <br> Answers should be given to 3 significant figures where appropriate. <br> Some questions are longer than others. <br> The number of marks for each question is shown in square brackets.

1. Inflation in Zimbabwe reached $25 \%$ per week in February this year. This means that prices increase by $25 \%$ over the course of a week.
a) By what number must I multiply the price of a newspaper to find its price one week later?
b) How much will an egg cost in one year's time ( 52 weeks) if its current price is 1 Zimbabwean dollar?
c) I can currently afford 10,000 loaves of bread with all my life savings (which I store under my bed). At this level of inflation, how many loaves of bread will I be able to afford in 6 months' time ( 26 weeks)?
d) How many weeks before my life savings will not buy a single loaf of bread?
2. Solve:
a) $\frac{x+2}{3}+\frac{2 x-1}{2}=\frac{1}{4}$
b) $\quad 2\left(x^{2}-3\right)=x(2-3 x)-5 x(4-x)$
c) $\frac{1}{x} \times \frac{2}{x} \times \frac{3}{x} \times \frac{4}{x}=\frac{3}{2}$
3. A path is built around a square area of grass with eight large identical rectangular paving stones, as in the diagram. The difference between the inside perimeter and the outside perimeter of the paved path is 16 m . The area of the grass in the centre is $42.25 \mathrm{~m}^{2}$.

Calculate the area of each paving stone.

[10]
4. a) A circular hose has internal diameter 12 mm and produces a jet of water with speed $8 \mathrm{~m} / \mathrm{s}$. It is used to fill a tank with a square base of side 2 m . Find how long to the nearest minute it would take to fill the tank to a depth of 1 m .
b) The hose is also used to fill a cylindrical tank. If the depth of water after 10 minutes is 1.2 m , find the base radius of the tank.
[ For a cylinder of base radius $r$ and height $h$, Volume $=\pi r^{2} h$.]
5. A right-angled isosceles triangle is partially covered by a square, so that two corners of the square lie on the hypotenuse, and one on each of the other two sides.

Calculate the proportion of the triangle that is covered by the square.
[Hint: Let the side of the square be 2 units.]

6. The diagram shows why the sum $1+3+5+7$ is equal to the area of a $4 \times 4$ square.

Use this idea to find the value of:
a) $1+3+5+7+9+11$
b) $1+3+5+\ldots+99$
c) $51+53+55+\ldots+2007$
d) $2+4+6+\ldots+2008$
e) $1+2+3+\ldots+2008$

[12]
7. ABCD is a tetrahedron, with all sides of length 2 . The base ABD lies flat.
a) $\quad \mathrm{M}$ is the midpoint of AB . Find the length CM .
b) $\quad \mathrm{N}$ is the midpoint of CD . By considering the triangle CMD, find the length MN.
c) By considering the area of triangle CMD, find the perpendicular height of C above the base ABD .
d) Hence find the volume of tetrahedron ABCD.

[For a pyramid with base area $A$ and perpendicular height h, Volume $=\frac{1}{3} A$.]
8. A regular hexagon has sides with lengths as marked in the diagram.

Find the values of $a, b$, and $c$, and hence find its perimeter.

9. The Fibonacci Sequence starts with the first two terms both being 1. The sequence continues, each term being the sum of the two previous terms, so the sequence begins $1,1,2,3,5 \ldots$

Work out what the remainder would be when the $2008^{\text {th }}$ term is divided by 7 .
[Note: you are not expected to work out all the terms! Find some way to investigate remainders when you divide by 7.]
10. The emblem of the National Bat Lovers' Association is in a square of side 2 . The large circle fits exactly inside a quarter-circle.
a) Find the radius of the large grey circle.
b) Find the area of the two wings (shaded black).

11. $\lfloor x\rfloor$ means "the largest integer less than or equal to $x$ ". e.g. $\lfloor 3.2\rfloor=3,[7]=7,\lfloor-3.4\rfloor=-4$.

Each on separate axes, with $x$ values from -4 to +4 , draw the following graphs:
a) $y=x-\lfloor x\rfloor$
b) $\quad y=\lfloor x\rfloor-2\left\lfloor\frac{1}{2} x\right\rfloor$
c) $y=(x-\lfloor x\rfloor) \times\left(\lfloor x\rfloor-2\left\lfloor\frac{1}{2} x\right\rfloor\right)$

